| Roll No. | | | | | | | |
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**24-ST-24**

# M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
## JUNE - JULY 2024

# STATISTICS
Paper - IV
## [Statistical Inference - I]

*[Max. Marks : 75]* *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

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**Note :** Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

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**Q. 1** What do you mean by point estimation ? How will you decide that a given estimator of a parameter is good ? Explain with suitable examples. **(15 Marks)**

**OR**

State and prove the invariance property of consistent estimator. Show that if T is an unbiased estimator of parameter $\theta$ , then $\lambda_1 T + \lambda_2$ is an unbiased estimator of $\lambda_1\theta + \lambda_2$, where $\lambda_1$ and $\lambda_2$ are known constants, but $T^2$ is biased estimator for $\theta^2$. **(15 Marks)**

**Q. 2** State and prove Cramer-Rao inequality and write regularity conditions. **(15 Marks)**

**OR**

**a)** State Lehman - Scheffe theorem and write its utility. **(05 Marks)**

**b)** State and prove Rao - Black well theorem. **(10 Marks)**

**Q. 3** Describe the method of maximum likelihood. A random variable X takes the value 0 and 1 with respective probability p and $1 - p$. On the basis of a random sample of size n, obtain the MLE of p. **(15 Marks)**

**OR**

Critically examine how interval estimation differs from point estimation? obtain 95% confidence interval for the mean of the normal distribution, when its variance is known. **(15 Marks)**

**Q. 4** Given **(15 Marks)**

$$f(x, \theta) = \begin{cases} 1/\theta & , \quad 0 \leq x \leq \theta \\ 0 & , \quad \text{otherwise} \end{cases}$$

and that you are testing the null hypothesis $H_0 : \theta=1$ , against $H_1 : \theta=2$, by means of a single observed value of $x$. What would b the sizes of type - I and type - II errors, if you choose the interval (i) $0.5 \leq x$ , (ii) $1 \leq x \leq 1.5$ as the critical regions ? Also obtain the power function of the test.

**OR**

State and prove the Neyman-Pearson Lemma for testing a simple hypothesis against simple alternative hypothesis. **(15 Marks)**

**Q. 5** Let $X_1, X_2, \ldots, X_n$ be a random sample from a N $(\mu, \theta)$ where $\theta$ is the unknown variance and $\mu$ is known. Obtain a likelihood and $\mu$ is known. Obtain a likelihood ratio test for testing a simple $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ **(15 Marks)**

**OR**

Explain how UMP test for simple null hypothesis against one sided alternative can be constructed. **(15 Marks)**

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